Acta Didactica Napocensia

Volume 8, Number 4, 2015

A STUDENT TEACHER'S CHOICE AND USE OF EXAMPLES IN TEACHING PROBABILITY[1]

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Abstract. The purpose of this study is to examine a mathematics student teacher's lessons by using example types named *teaching concepts and procedures* and *the provision of exercises*. The participant's two lessons regarding the probability concept were observed and the semi-structured interviews were realized. The participant preferred to give examples taken from real world as a part of teaching concepts and procedures aiming at concept formation and provision of exercises.

Keywords: knowledge quartet, choice and use of examples, secondary mathematics student teacher, probability.

Introduction

In studies related to "subject matter knowledge" (SMK) and "pedagogical content knowledge" (PCK) of mathematics teachers, "Knowledge Quartet" (KQ) has been used as a model which helped to evaluate and to improve both SMK and PCK (Huckstep, Rowland, and Thwaites 2006; Petrou, 2009; Rowland 2005, 2007; Rowland, et al. 2009; Rowland, Huckstep, and Thwaites 2003, 2005; Rowland and Turner 2007; Turner 2007). KQ consists of four units and one of them is named transformation. The transformation requires the use of models, analogies, metaphors, examples, representations, and demonstrations which can become a bridge between the teachers' understanding of the subject and the understanding that the students are expected to gain (Uşak, 2005). Transformation includes the presentation ways in which the teacher's own knowledge is transformed to make it accessible to the students (Turner, 2007). This unit also includes the selection of examples and procedures used to assist concept formation, the use of multiple representations and presentations, as well as focusing on planning preparation and conducting the teaching process (Rowland et al. 2009; Rowland, Huckstep, and Thwaites 2003, 2005; Thwaites, Huckstep, and Rowland 2005). Petrou (2009) defines transformation as the knowledge-in-action and indicates that this unit includes the representations and examples used by teachers, as well as, the teachers' explanations and questions asked to students throughout the teaching. The transformation unit has three codes; one of them is named "choice and use of examples" (Rowland, Huckstep, and Thwaites 2005).

Example is defined as specific instantiation of a general principle, chosen to illustrate or explore that principle (Chick, 2007). Bills and Watson (2008) state that examples have always played a central role in both development of and teaching of mathematics. Examples are significant when demonstrating the procedural and conceptual understanding in addition to finding the relationship and making generalizations (Suffian and Abdul Rahman, 2010). It is observed that the use of examples is

[1] This paper is made up of an improved version of the poster presentation presented at the 35th Conference of the International Group for the Psychology of Mathematics Education, Middle East Technical University, 10-15 July 2011, Ankara, Turkey.

Received December 2015.

examined by different researchers under different categories. Rissland-Michener (1978) distinguishes four categories: (a) start-up examples that draw attention to the principle, (b) reference examples that are standard occurrences frequently referred to in the general theory, (c) model examples that show the typicality of a situation, and (d) counter-examples that show conditions under which the general principle might not apply (cited in Chick and Harris, 2007). Metaxas (2008) states that the examples used are examined under various categories in terms of pedagogical distinctions. Metaxas (2008) distinguishes these categories as; examples and illustrations (Sowder, 1980); examples and exercises (Renkl, 2002); generic examples, counter-examples and non-examples (Bills, Dreyfus, Mason, Tsamir, Watson, and Zaslavsky, 2006); specific, general and semi-general counter-examples (Peled and Zaslavsky, 1997). Rowland and Zaslavsky (cited in Bills et al., 2006) propose two categories; (a) the examples that are used to illustrate a general principle along with reasoning and (b) examples that are provided as exercises for practice. Chick and Harris (2007) emphasize that the teachers use examples (a) to initially illustrate concepts and (b) for students to work on during the consolidation process. Rowland et al. (2009, p. 70) state that mathematics teachers use examples with the aim of "teaching concepts and procedures" and for "the provision of exercises". It is stated that the types of examples discussed in terms of teaching concepts and procedures are used to make the acquisition of a concept or operation easier and it is emphasized that it is better to start a subject with examples before giving the definition (Rowland et al., 2009). It is declared that the examples discussed in the context of the provision of exercises are used to consolidate and apply new thoughts or methods (Rowland et al., 2009). In addition to this, it is known that there has been limited research done on the teachers' choice of examples (Bills, Mason, Watson, and Zaslavsky 2006). It is also observed that there is limited information in the studies about the mathematics teachers' and mathematics student teachers' use of examples in their teachings, regarding what their choice of examples depends on, what kind of choice they make, and how their choices affect their teaching. So in this context, any study which will be carried out about the teachers' and the student teachers' choice of examples and its reflection on their teaching is considered to be important in the effort to reveal the different aspects regarding choices of examples. This study focuses on a student teacher's choice and use of examples. The purpose of this study is to examine a mathematics student teacher's two lessons in the context of "choice and use of examples" for the concept of probability. Her examples were examined in the framework for example types of Rowland et al. (2009) and the answer to the following question in this article was attempted for research purposes:

How was the probability teaching of a mathematics student teacher shaped in terms of the example types named "teaching concepts and procedures" and "the provision of exercises"?

Method

The qualitative case study design was used in this research. Because the aim of the study analyzed choice and use of examples of one participant in the context of framework of Rowland et al. (2009) about example types in a detail way, the case study design was selected. In this study, the participant's probability teaching was examined in terms of choice of examples and their reflections on the teaching in her two probability lessons.

Participant. The participant is one senior student teacher who was studying at secondary school mathematics teacher education program. The participant took part in the research voluntarily. The real name of the student teacher was not used in the research; the pseudonym (Ege) was used when giving information about herself and while the analysed data were presented. In our country, the secondary mathematics student teachers are educated with five year thesis excluding master programs. In these programs, SMK, general pedagogical knowledge, and PCK oriented lessons are taught. In the last three semesters of these programs, there are courses related to school-based placements named School Experience I- II, and Teaching Practice. Ege also had teaching experience in a high school in the USA in the scope of the Fulbright scholarship program.

Ege's students were taught the concepts of sets, numbers, functions, permutation and combination in the high school prior to the probability teaching. Additionally, they were educated on the basic topics regarding probability at the end of the primary education. There were a total of 21 students in Ege's class. These students were taught their previous mathematics lessons traditionally. In these lessons,

their teachers gave the definitions and made them solve exercises once the definitions were understood. Ege paid attention to making the students more active, creating classroom discussion and using examples before giving the definitions concerning the topic. The following statements of Ege showed that she adopted the constructivist approach as well as paying her attention to the use of real life situations and the concept formation activities in her lessons:

> I think mathematics serves us everywhere in our lives. So I want to share my knowledge for my current and future ability. I also want my students to see that mathematics is important for their lives. The reason why I wanted to be a mathematics teacher is that mathematics is the only thing I treasure much in my life despite it being a lesson. Also to make all students have my desire and sympathy about mathematics is an effective reason… I am confident about myself with regards to teaching mathematics. To be a good mathematics teacher, I try to enhance my mathematics knowledge as well as my teaching knowledge to the highest point. I can make up my deficiencies by means of my experiences instead of reading theoretical instructions. My teaching experiences will contribute to teaching mathematics. I like preparing concept formation activities and seeing my students form the concepts step-by-step. (Interview prior to the lesson)

Instruments. Data were collected from the lesson plans which were prepared by Ege for the concept of probability (see Appendix 1, Appendix 2), 2 hour video records of her lessons and the voice record of the semi-structured interviews before and after her teaching.

Data Analysis. Video records of the participants during the process of teaching the concept of probability were watched numerous times by researchers. While analyzing the video records of the lessons in the research, descriptive summaries were written. While the data were being analyzed, "choice and use of examples", which is one of the KQ codes, was used and the frequency of facing example type categories in the lesson was determined by using content analysis. The obtained findings are presented in Table 1. The example type categories in question were described by Rowland et al. (2009) as teaching concepts and procedures and the provision of exercises. In the table, the names of each example type are given in the columns and the findings about the lessons are indicated for each lesson in the columns of Lesson 1 (L1) and Lesson 2 (L2) by giving example numbers (E1, E2, …, E12) to these findings. The names of these examples are listed below:

E_1: The example of the numbers with no repeating digit which can be formed with the elements of a set

E_2: The example of the number of different events that can occur in a set

E_3. The example of blue eyes

E_4: The example of fair coin

E_5: The example of height

E_6: The example of shoe size

E_7: The example of Facebook activity

E_8: The example of classroom list

E_9: The example of super league

E_{10}: The example of fair dice

E_{11}: The example of marbles

E_{12}: The example of draw.

To make the findings of the study clearer, the authors used extracts from the video records of the lessons. These records included the statements of the participant and her students, the screenshots of the presentations projected by Ege, and the information that Ege and her students wrote on the board. The voice records of semi structured interviews carried out with Ege were transcribed. These data were used to reflect her views regarding the choice and use of examples.

Findings

In this study Ege's choice and use of examples on the probability teaching was analyzed and Table 1 was created to show the examples type for twelve examples within her two lessons. In addition, for each example, it is shown in Table 1 how long it took in a 45 minute class hour in parentheses.

Table 1. The analysis of the lessons of Ege in the context of use of examples

	teaching concepts and procedures (min)	the provision of exercises (min)
Lesson 1	$E_7(20)$	$E_1(4)$-$E_2(3)$-$E_3(2)$-$E_4(2)$-$E_5(1)$-$E_6(2)$
Lesson 2	$E_8(2)$-$E_9(2)$	$E_{10}(5)$-$E_{11}(7)$-$E_{12}(3)$

Ege used two types of examples in her teaching and it was determined that Ege made a choice of examples in the context of teaching concepts and procedures three times and the provision of exercises nine times. Ege spent the most time on E_7-a totally 20 minutes which was an example in the context of teaching concepts and procedures. Ege explained her interview the reason why she spent 20 minutes on E_7 as follows:

> As Facebook is an activity intended to teach a concept, I spent more time on it. The other examples were aimed for the students to generally apply what they had learned. For this reason, those examples didn't occupy much time. (After the lesson)

In her first lesson, Ege started an intellectual discussion to make an introduction to the subject and asked her students what the concepts of experiment, output, sample space, sample point, space, certain event, discrete event and impossible event, which they had learned at previous class, meant. Afterwards she asked them to give examples from their daily life related to these concepts. For instance; the students gave the examples of elephants' flying and that when the dice was thrown, the number 7 was rolled for the impossible event; and for the certain event that the green plants photosynthesize and an even numbers follows an odd numbers. In this process, Ege made the students discuss these concepts in the classroom and find examples for the each concept. It was observed that the students did not face any challenges during this process. Ege explained the reason why she started the lesson by asking questions to the students as follows:

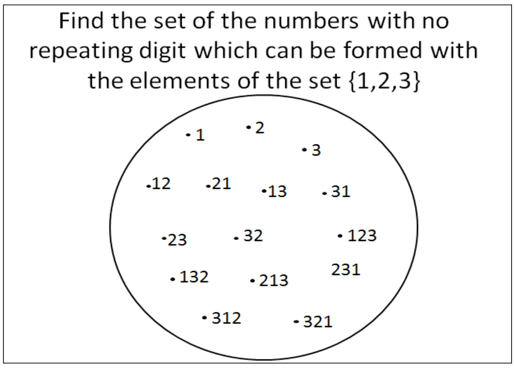
> The students already had the pre-knowledge about these concepts. So I wanted them to remember their knowledge about these concepts by giving examples for them. (After the lesson)

Ege indicated in her lesson plan that she would ask the students questions (see Appendix 1) about the so-called concepts that she wanted them to discuss. She stated that while she was preparing her lesson plan and especially the examples in her lesson plan, she used different textbooks as well as the activity examples that were in the curriculum. Ege stated that she chose the examples from E_1 to E_6 for the purpose of making students remind themselves of their pre-knowledge, warm up the subject and review the basic concepts of probability as follows:

> I wanted to remind student of the preliminary concepts which are necessary for the learning of the concept of probability function using my activities up to the Facebook example. Therefore, I planned to fulfill the students' lack of knowledge by making students solve questions about the pre-subjects. I think that I will prevent the problems from appearing about the students' pre-knowledge in the Facebook activity. (Before the lesson)

Ege, who taught her lessons in the direction of her lesson plan, gave *the example of the numbers with no repeating digit which can be formed with the elements of a set* (E_1) in the context of the provision of exercises. E_1 has been discussed in the context of the provision of exercises because it was an example that students will repeat their pre-knowledge about the concept of permutation, will reveal their pre-knowledge about the concept of probability and will resolve their deficiencies if needs be. Ege asked students the question "Find the set of the numbers with no repeating digit which can be formed with the elements of the set {1,2,3}".After asking this question, the answers arose from the students about the all numbers which would be composed of one-digit, two-digit and three-digit

numbers with no repeating digit. After the students said the all numbers composed, Ege showed her students the set formation in her presentation as follows.



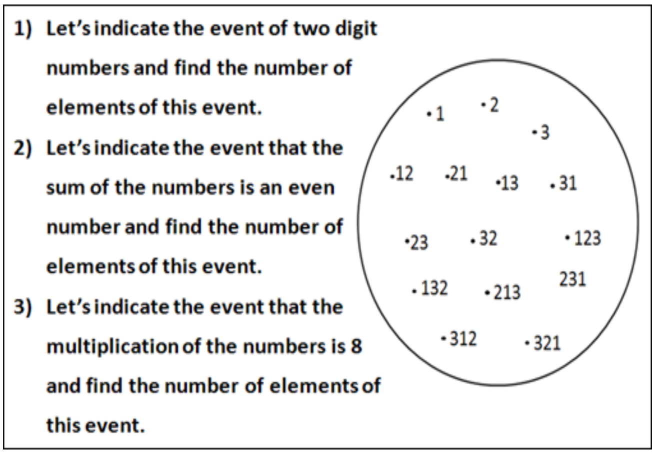
Ege asked the example E_1 to the whole class and made her students find the answer first. Then Ege called a willing student to the board and asked him to solve it on the board. The student who came to the board gave a wrong answer but Ege did not point out his mistake and by asking questions, she tried to make him correct his mistake. As previously stated in the interview, Ege realized the student's deficiency, did not say the answer directly and waited him to correct his mistake. So the example chosen in the context of provision of exercises reached its goal and contributed to resolve the student's deficiency.

Board:	
	$s(E) = \binom{3}{1}\binom{3}{2}\binom{3}{3}$
Ege:	Can you explain what you are doing here?
Student:	One digit number plus two digit number plus three digit number.
Ege:	Yes.
Student:	And can we do it like that?
Board:	
	$2^3 + 2^2 + 2^1$
Ege:	Do you think that we can?
Student:	13(*The student calculates this sum*). No we can't.
Students:	Shouldn't they be summed?
Ege:	Then why isn't it a multiplication but a plus?
Student:	Because it's a discrete event.
Ege:	Umm, then what is a discrete event?
Student:	It means an event which doesn't have intersection.
Ege:	Ok (*adding a plus sign instead of multiplication*).
Board:	
	$s(E) = \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$

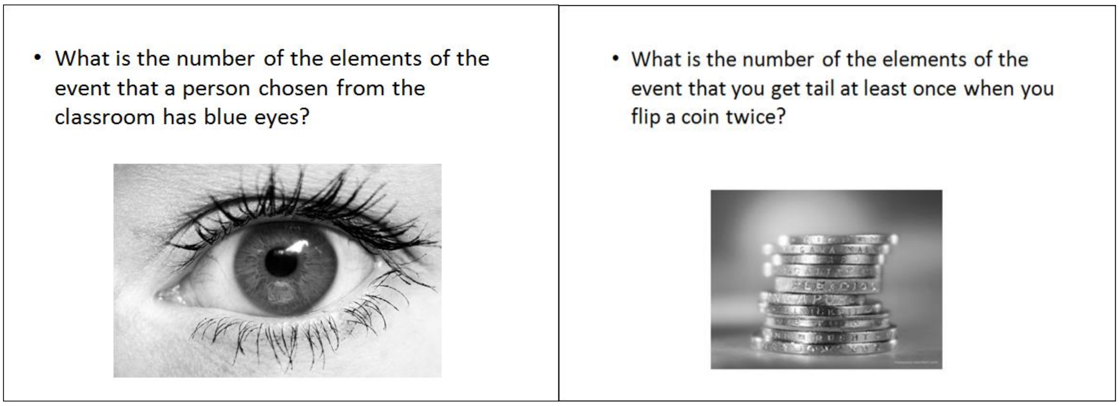
Ege stated lesson plan that she added different questions to E_1 as an exercise for the practice, output and sample space which she asked the students to make a definition of and give an example to. In this example, the questions "What is an experiment? What are the outputs? What is sample space? How many elements does it have?" were asked to the students.

In a similar way, by using *the example of the number of different events that can occur in a set* (E_2) in the context of the provision of exercises, Ege aimed to help her students to consolidate the subject that they had discussed. For the example in question, Ege told the students; "Let's indicate the event of two digit numbers and find the number of elements of this event. Let's indicate the event where the sum of the numbers is an even number and find the number of elements of this event. Let's indicate the event where the multiplication of the numbers is 8 and find the number of elements of this event." Ege tried to create different events in a given set thanks to these questions. The students chose the elements

which were appropriate to the event benefiting from this set and identified the number of the elements of the event in question. They explained the answer of these three questions verbally by using the set.



Ege used the examples E_3 and E_4 to emphasize the importance of the number of elements and consolidate the knowledge of the students about the sample space. While Ege was using the example E_3, she first discussed it with her students and then formed a set that had the names of the students who had blue eyes and wrote the number of the elements of the set on the board. In the example of E_4, she called a student to the board to solve the example. Ege displayed her approaches on the set which she formed using real life situations from E_3 to E_6. These approaches have been used for the set formation with the numbers in E_1 and E_2. Ege would have applied the examples E_3, E_4, E_5 and E_6 which were more meaningful, concrete and clear for students before E_1 and E_2.

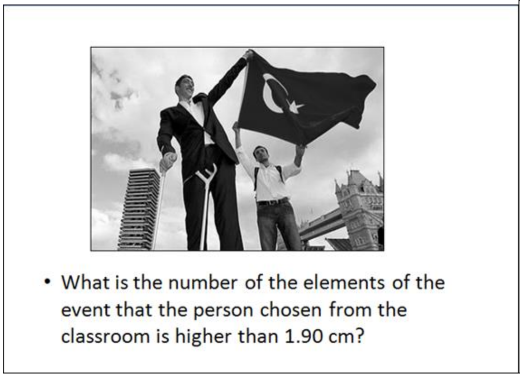


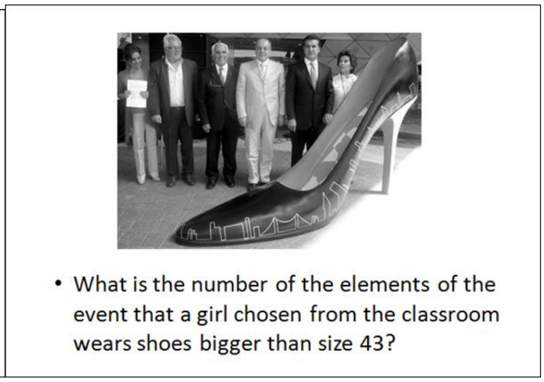
When Ege asked who the blue-eyed students in E_3 were, she understood that there were no blue-eyed students in the class. Then she asked who the green-eyed students in her class were and she explained this adjustment as follows:

> I did not want to use this example as it was an example of the impossible event. I thought at least one student whose eyes were blue would be in the class. When there was no blue-eyed student. I promptly learned that there were green-eyed students in the class, so I changed the question. (After the lesson)

Another examples which Ege used in the context of the provision of exercises was *the example of height* (E_5) and *the example of shoe size* (E_6) by which she tried to exemplify the impossible event. In E_5, Ege asked the students to find the number of the elements of the event that the student chosen from girls in the classroom was taller than 1.90 cm. After an exchange of ideas in the classroom, they stated that there was no girl who was taller than 1.90 cm in the classroom and that this event was impossible. With an approach similar to the previous example in E_6, Ege asked the students to find the number of

elements of the event that a girl chosen from the classroom wore shoes which were bigger than a size 43. Ege wanted the students to realize that there were no girls in the classroom who wore a size 43 shoe and that this was also an impossible event.





Ege explained the reason why she preferred to give examples over the students chosen from the classroom in E_3, E_5 and E_6 as follows:

> I used different examples while teaching the lesson to make students comprehend the concept. At the beginning of the lesson, I asked some questions. I tried to give examples taken from the class as much as possible to make them comprehend the concept of sample space. For instance I asked them whether there were any girls in the classroom who was wearing 43 size shoes to make the concept of impossible event meaningful. I think this was successful because instead of saying "No there is not" they said impossible event. (After the lesson)

Ege tried to engage students by promoting her examples chosen from daily life with pictures. In particular she made her students join in the discussion so that she had a strong interaction with them. Furthermore, the tall man in E_5 is a well-known person in Turkey and he was introduced as the tallest living person in the world. When her students saw this man on the slide, they said his name and paid attention to the example shown to them. In this sense, it is thought that using striking examples are important and successful.

After Ege reminded the students of their pre-knowledge and made them solve the examples as exercises to consolidate their knowledge, she gave the students *the example of Facebook activity* (E_7). E_7 was the first example that Ege used in the context of teaching concepts and procedures. Ege began to handle the concept of probability with this activity for the first time. Before Ege gave the fundamental structure and the definition of the probability function, she made them complete the Facebook activity and hoped that they would reach the knowledge that the probability was a function of them (see Appendix 1). Ege explained why she designed an activity about Facebook before the lesson as follows:

> Facebook has become a very influential social network especially for teenagers in the current years. Almost all students have a Facebook account and they spend most of their time using it. So I find it necessary to prepare an activity about Facebook to make the lesson more appealing. (Prior to the lesson).

In this activity, the probability that the chosen person opened a Facebook account to make new friends was determined as P(a)=0.2, to find old friends as P(b)=0.1 and to share video, pictures etc. as P(c)=0.7. After that, Ege made a statement that "The probability of opening more than one account of somebody chosen is equal to the sum of these probabilities separately". Ege then asked the students to write the probabilities of all the events by taking the sample space and its subsets into consideration.

- **Please write the sample space and its subsets.**

written by a student

$E = \{a, b, c\}$

$A = \{\{\ \}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

- **Please write the all probabilities of the situation regarding given data using the subsets of the sample space.**

Ege gave the students activities to be completed individually. While she was carrying out the activity, she called a student to the board to solve the problem. The student corrected the mistake which he made while writing the mathematical expressions himself after he was warned by Ege.

Board: $E = \{a, b, c\}$

$A = \{P(0) = 0, \ldots\}$

Ege: But now you wrote the probabilities (*the student corrects his mistake*).

Board: $E = \{a, b, c\}$

$A = \{\{\ \}, \{a\}, \{b\}, \{c\}\{a, b\}, \{a, c\}, \{b, c\}\}$

Ege wanted the students to find out the probabilities of all events in addition to the subsets of the sample space and to show it on the slide. After her students wrote the probabilities of the subsets in question, Ege expected them to find the highest and lowest ones among these probabilities and make a connection between the results and the concepts that they discussed at the beginning of the class.

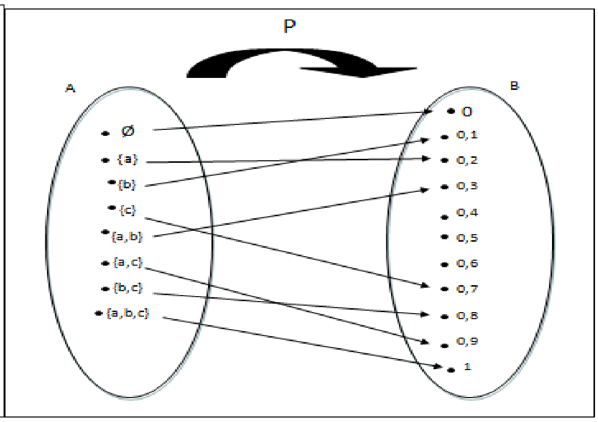
• **P(Ø)=** • **P({a})=** • **P({b})=** • **P({c})=** • **P({a,b})=** • **P({a,c})=** • **P({b,c})=** • **P({a,b,c})=**	• **Please find the highest and lowest ones among these probabilities.** • **Please make connections between these and the concepts that they discussed at the beginning of the lesson.**

Before Ege gave information about the probability function, she asked her students to match the subsets of the sample space and its probabilities. The students associated the probabilities of $P(\emptyset) = 0$ ve $P(a, b, c) = 1$ with the impossible event and the certain event. Ege ended the debate by explaining that the probability of impossible event was 0 and the probability of certain event was 1. In the next slide, Ege asked the students which concept this showed a similarity and she gained the concept of function as an answer from the students. Then she asked the students to make the definition of the function in question and find its domain and range. Ege indicated the domain and range of the probability function and then made a match between them. Thus Ege allowed her students to reach the

end of the probability function by guiding her students. In this process, she helped them to relate their pre-knowledge regarding the concept of function with probability function.

- **Please perform the following matches according to the results that you found.**

{ }	**0**
{a}	**0.1**
{b}	**0.2**
{c}	**0.3**
{a,b}	**0.4**
{a,c}	**0.5**
{b,c}	**0.6**
{a,b,c}	**0.7**
	0.8
	0.9
	1



Ege called different students to the board to solve the problems and she also created a discussion environment in the classroom during the Facebook activity. After Ege did the Facebook activity with her students, she ended her first lesson by giving the definition of the probability function.

- The probability function **is a function defined from the subsets of the sample space to the interval of [0,1] and verify the axioms as follows.**
- **When A is an event in the sample space E;**

$$0 \leq P(A) \leq 1$$

$$P(E) = 1$$

$$A \cap B = \emptyset \quad ise \quad P(A \cup B) = P(A) + P(B)$$

Ege started her second lesson by reminding students of the topics taught in her first lesson. Ege made a choice of examples in the context of teaching concepts and procedures two times and provision of exercises three times in her second lesson (see Table 1). She used two examples from daily life to help the students understand what equiprobable sample space meant. In *the example of classroom list* (E_8) which was in the context of teaching concepts and procedures, she wanted the students to discuss the probability of any student being chosen from the list and the equality of this probability for each student. The students answered that the probabilities could not be equal for Ege's second question. Ege then lead students towards the right answer. She stated that the person chosen first would be added to the classroom list again for a second chance. So her students stated the probabilities would be equal under the condition said by Ege when she asked the question again. The following example of her was about the equality of the probabilities. In *the example of super league* (E_9), the students talked about the probability of any soccer team being the champion in the super league.

- What can you say about that any of you is chosen from the classroom list?
- Is the probability that each of you can be chosen equal?

- Super League is about to start and one hour is left for the first games to start. All the teams have new players transferred and have completed all their works.
- As a sports critic, by looking at the teams objectively what can you say about the probability of each team' being the champion?

Ege's example regarding the super league in E_9 created a discussion environment in the classroom. Some students expressed that the probability of being champion for each team could not be equal to each other because they thought the championship in the super league was impressed by too many variables. This example caused the students to be slightly confused. For the purpose of demonstrating the equality of probabilities, Ege could fictionalize her example as not to be impressed by different variables. An extract from Ege's lesson is as follows:

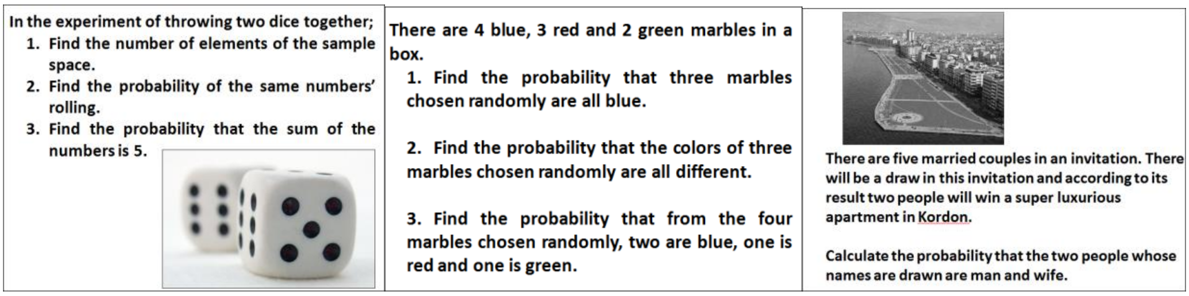
Student: $\frac{1}{18}$ (*There are 18 teams in the super league*).
Ege: Not numerical.
Student: Then all of them are equal.
Ege: All of them are equal. For example, is the probability of being champion of Bucaspor (*recently rising in the super league*) and Galatasaray (*the best soccer team in Turkey*) equal?
Student: Not equal (*laughing*)
Ege: But the league has recently started. I'm asking you the probabilities of the matches being played.
Student: Equal.
Student: Not equal.
Ege: Isn't it equal? In the end, there is the probability of being champion for all teams, isn't there?
Ege: Well, we are choosing a person from your class or a team from the super league now. When we choose any person or look at the probability of being champion for a team, these probabilities are equal, aren't they? Now let's make a generalization! What can we say? For any team?
Student: The probability of being chosen every subset in a universal set is equal.
Ege: It is right if all of them are independent and separate from each other of course.

Ege aimed to reach the equiprobable sample space through the examples E_8 and E_9 and throughout these examples she created a discussion in the classroom. After Ege discussed the two examples of equiprobable sample space in the context of teaching concepts and procedures with the students, she gave the definition of the equiprobable sample space. It was observed here that prior to forming the concept, Ege used examples related to this concept.

- **The sample space whose every sample points are equal to each other is called "equiprobable sample space".**
- **The probability of an event A;**

$$P(A) = \frac{s(A)}{s(E)} = \frac{Number\ of\ the\ elements\ of\ event\ A}{Number\ of\ the\ elements\ of\ event\ E}$$

Ege made the students solve the examples in the context of the provision of exercises. In *the example of fair dice* (E_{10}), she asked the students to indicate the number of the elements of the sample space, to find the probability of the same numbers rolling and the probability that the sum of the two numbers was 5 in the example of throwing the dice. First Ege wanted her students to consider this example and then she collected their answers and opinions concerning the so-called questions. In *the example of the marbles* (E_{11}), Ege asked the students to calculate the probability that the different colored marbles in a box were chosen (see Appendix 2). Ege wanted her students to answer the questions in her example too. In the end they solved the questions on the board and explained their solutions to other students. Ege aimed to consolidate the knowledge that her students had learned about the probability function using this example. In *the example of draw* (E_{12}) which was another example in the context of the provision of exercises, Ege asked the students to find the probability where the winners in a draw for a super luxurious apartment were a married couple (see Appendix 2).



While using the examples E_{10}, E_{11} and E_{12} in her teaching, Ege called different students to the board for each example and made them explain how they came to solve the examples to their classmates. Ege expressed her ideas about how she first helped the students comprehend the equiprobable sample space and then answered these three questions to consolidate their knowledge of this concept as follows:

> First of all, by giving two examples I tried to help the students reach to the concept of equiprobable sample space. Then I prepared exercises about this concept. I thought that I should ask them three main questions instead of numerous questions. I wanted to ask multiple choice questions and I wanted these questions to be answered step by step. (After the lesson)

After Ege expressed about her examples, she asked the students to work in groups and prepare their own examples in the context of the provision of exercises and also solve these examples. She then explained that she would give the examples that she had mixed to the each group and that the group whose example could not be solved would be the winner. The students also solved the examples which they had prepared. However the lesson ended before the examples could be given to the groups. Ege explained in the interview the reason why she created a contest and why she made the students design their own questions as follows:

> After the exercises about the equiprobable sample space, I could have asked more comprehensive questions, but I thought it would be different to organize a contest so I did. I separated the class into groups and made them prepare probability questions in the last 15 minutes of the lesson. I told them to write probability questions about the subject and the concepts that we had talked about in the lesson.
> *The researcher*: Why did you plan something like that?
> I planned this because if they learn this concept they can define and exemplify it. They can also write questions related to it. I tried to evaluate this. Did they comprehend the concept or not? And I also thought it would possibly create a competitive environment which could be enjoyable.

Conclusion, Discussion and Suggestions

In the result of identifying the participant's two-hour probability instruction in the context of choice and use of examples, it was generally observed that she preferred giving examples from real world situations as a part of teaching concepts and procedures aiming at concept formation. On the other hand, she gave examples in the context of the provision of exercises aiming at practicing those concepts. In the first lesson, the participant wanted her students to remember the concepts (experiment, output, sample space, sample point, space, certain event, discrete event) which they had learned prior to this lesson and to give examples related to those concepts. Following this, she used exercises to make sense of those concepts. In the interview, she stated that her aim in preparing the Facebook activity was to relate with the real world and the function concept learned in previous years and the probability concept. In the second lesson, she gave examples from the real world to introduce equiprobable sample space and she presented three exercises aiming at familiarization and practice. Before the participant gave the definitions, she chose and used the examples for her students to reach the concept using their pre-knowledge. She used the examples in the context of teaching concepts and procedures relevantly. After her students reached the concept and once she gave the definition, she gave place to the examples in the context of the provision of exercises and aimed to consolidate the concept. Rowland et al. (2009) emphasized that use of two types of examples could be for these purposes.

The participant carried out her lessons using two types of examples. It was seen that the participant's choice and use of examples was generally appropriate. However, using the example of super league caused the students to become confused. It is important that the examples used about the equality of probabilities should be given for the events which cannot be affected by different variables. Besides when the example of super league is generally male appropriate, it will not be appealing for the girls. When this point of view is taken into consideration, it is important that the choice of real life situations is appropriate for both genders. The examples used which can be understood by students in their daily lives and which are fictionalized well will be more affective to reach its purpose.

The participant stated that she prepared the examples with the use of textbooks and the curriculum. In a similar way, in the study carried out by Yusof and Zakaria (2010) the participant stated that secondary school mathematics teachers made choices of intense and repetitive examples and found these examples from textbooks or exam questions. However in this study it was observed that the participant used different types of examples instead of repetitive examples. It was observed that these examples were also interesting for the students and did not bore them. In addition to this, the participant arranged the examples that she had prepared using different sources in the direction of the acquisitions regarding the concept of probability in the national mathematics curriculum and taking the degree of difficulty into consideration. In the study which was carried out by Rowland, Thwaites and Huckstep (2003), it was observed that the teachers needed guidance and help about the use and choice of examples in mathematics teaching as they had difficulty in choosing these examples.

In the interview, the participant emphasized that in the process of preparing her lesson, she paid attention to the factors about the choice of examples which she did not consider in previous lessons (about the limit concept). In the study which was carried out by Turner (2007), the observations regarding the instructions of the student teachers, participant group meetings and the ideas that the participants wrote about the instruction constituted the data of the study. In the consequence of the reflections carried out through KQ, it was observed that in their second year, the participants paid most attention to the factors which they did not pay attention in their first year (Turner, 2007). In the interview which was carried out at the end of the first year, the participants stated that they paid more attention to the choice of examples, making associations, and creating an environment for the students in which they could develop their own strategies etc (Turner, 2007). In this context, evaluating the instruction by using KQ is thought to make contributions to student teachers in the choice and use of examples. It is considered that in the process of teaching, it is important to use the examples in the context of both teaching concepts and procedures and the provision of exercises. It is recommended that the examples in the context of teaching concepts and procedures should be used more frequently as they play an important role in the process of comprehending the concepts and procedures. Therefore, it will be easier to apply the acquired knowledge in the examples for the context of the provision of exercises.

Attention should be given to the importance of choices and uses of examples in the training of student teachers. Student teachers should be informed how the examples in the context of teaching concepts and procedures and the provision of exercises should be used in the lessons.

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Appendix 1

The Instructor	: Deniz
Lesson	: Mathematics
Grade	: 10th Grade
Class Hour	: 45'
Date	: 31.05.2010
Subject	: Probability
Acquisitions	1. Explain the concepts of experiment, output, sample space, sample point, space, certain event, discrete events. 2. Calculate the probability of an event by indicating the probability function and show the basic characteristics of the probability function.
Materials	: Power point presentation, sheets of exercises

Time	**Activity of the Instructor**	**Activity of the Students**
3'	Asks the students the meanings of the concepts of experiment, output, sample space, sample point, space, certain event, discrete events and impossible event. Asks them to give examples to these concepts from daily life.	Answers the questions that the teacher asks and gives examples to the concepts.
10'	With the aim of exemplifying the above mentioned concepts, the instructor forms a set of all numbers with no repeating digits which can be formed with the elements of the set {1,2,3}. She asks questions about this set: • What is an experiment? • What are the outputs? • What is sample space? How many elements does it have? • Explain the event of two digit number and find the number of the elements of this event. • Explain the event that the sum of the numbers is an even number and find the number of elements of this event. • Explain the event that the multiplication of the digits is 8 and find the number of elements of this event.	Answers the questions that the teacher asks.
3'	With the aim of emphasizing the importance of the number of the elements of the events, she asks the students questions from daily life and from classroom: • What is the number of the elements of the event that a person chosen from the classroom has blue eyes? • What is the number of the elements of the event that you get tail at least once when you flip a coin twice? • What is the number of the elements of the event that the person chosen from the classroom is higher than 1.90 cm? • What is the number of the elements of the event that a girl chosen from the classroom wears shoes bigger than size 43?	Answers the questions that the teacher asks.
12'	Teacher gives the sheets of exercises that she has prepared to the students. Teacher reads the scenario of Facebook that she has prepared. After giving necessary information, she reads the directive questions to the students and expects them to give answers. She calls the students to the board to answer the questions. She helps the students to understand that the matching carried out after the questions is a function.	Reads the scenario on the sheet of exercises. Reads the directive questions one by one and tries to answer the questions. Goes to the board to solve the example.
5'	Teacher asks what are the general characteristics of the function whose definition and range have been given.	Tries to guess the general characteristics of the function whose definition and range have been given.
3'	Teacher gives the definition and the characteristics of the probability function.	Listens to the definition of the teacher, takes notes and asks the teacher when they do not understand.

Appendix 2

<table>
<tr><th colspan="3">LESSON PLAN</th></tr>
<tr><td colspan="3">The Instructor : Deniz
Lesson : Mathematics
Grade : 10^{th} Grade
Class Hour : 45’
Date : 03.06.2010
Subject : Probability</td></tr>
<tr><td colspan="3">Acquisition
3. Explains the equiprobable sample space and states that $P(A) = \frac{s(A)}{s(E)}$ for an A event in space.</td></tr>
<tr><td colspan="3">Materials : Power point presentation, sheets of exercises</td></tr>
<tr><td>Time</td><td>Activity of the Instructor</td><td>Activity of the Students</td></tr>
<tr><td>8’</td><td>Asks some questions from daily life to make the students understand what the equiprobable sample space means:
• What can you say about that any of you is chosen from the classroom list? Is the probability that each of you can be chosen equal?
• Super League is about to start and one hour is left for the first games to start. All the teams have new players transferred and have completed all their works. As a sports critic, by looking at the teams objectively what can you say about the probability of each team’ being the champion?</td><td>Tries to guess the answers of the questions that the teacher asks.</td></tr>
<tr><td>2’</td><td>Gives the definition of the equiprobable sample space.</td><td>Listens to the teacher and takes notes.</td></tr>
<tr><td>20’</td><td>Asks the following questions to consolidate the concept:
• In the experiment of throwing two dice together;
Find the number of elements of the sample space.
Find the probability of the same numbers’ rolling.
Find the probability that the sum of the numbers is 5.
• There are 4 blue, 3 red and 2 green marbles in a box.
Find the probability that three marbles chosen randomly are all blue.
Find the probability that the colors of three marbles chosen randomly are all different.
Find the probability that from the four marbles chosen randomly, two are blue, one is red and one is green.
• There are five married couples in an invitation. There will be a draw in this invitation and according to its result two people will win a super luxurious apartment in Kordon. Calculate the probability that the two people whose names are drawn are man and wife.
Asks the students to come to the board to solve the examples.</td><td>Tries to solve the questions that the teacher asks in the desk and then goes to the board to show his/her friends how she/he has solved it.</td></tr>
<tr><td>15’</td><td>Organizes a contest which has the following instructions:
• Separates the class into groups.
• Asks each group to write a problem about probability.
• After 5 minutes, she collects the questions and gives them back to the students after mixing them.
• She gives 5 minutes for each group to solve the question that they have taken.
• The group whose question cannot be solved is announced to be the winner and a student from each group goes to the board to solve the question.</td><td>Listens to the directive questions of the contest, writes their question as a group, after getting the question of the other group, starts solving the new question.</td></tr>
</table>